The Normal Distribution

Worksheet 2 — Set B

Mathematics: Analysis and Approaches HL

Instructions

- Answer all questions in the spaces provided. Show all working clearly: correct answers without working may not receive full marks.
- Unless told otherwise, give numerical answers exactly or correct to three significant figures.
- Questions are arranged in order of increasing difficulty. Question 10 on each worksheet is a *challenge question*, set beyond typical examination level.
- A calculator may be used unless a question indicates otherwise.

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1. [Maximum mark: 4] *Difficulty:* 1/5

The random variable $X \sim N(55, 7^2)$.

(a) Find $P(X < 48)$. **[2]**

(b) Find $P(X > 65)$. **[2]**

2. [Maximum mark: 5] *Difficulty:* 1/5

The random variable $X \sim N(80, 12^2)$. Find the value of a such that $P(X < a) = 0.20$. **[5]**

3. [Maximum mark: 5] *Difficulty:* 2/5

The random variable $X \sim N(\mu, 6^2)$. Given that $P(X > 40) = 0.25$, find the value of μ. **[5]**

4. [Maximum mark: 6] *Difficulty:* 2/5

The random variable $X \sim N(45, 8^2)$. Find $P(35 < X < 55)$. **[6]**

5. [Maximum mark: 7] *Difficulty:* 3/5

The diameters of bolts produced in a factory are normally distributed with mean 50 mm and standard deviation 2 mm. A bolt is graded “oversized” if its diameter exceeds 53 mm.

(a) Find the probability that a randomly selected bolt is graded oversized. **[2]**

(b) In a random sample of 15 bolts, find the probability that exactly 2 are graded oversized. **[5]**

6. [Maximum mark: 8] *Difficulty:* 3/5

A bakery produces two types of loaf. Type A loaves have mass M_A grams, where $M_A \sim N(45, 3^2)$. Type B loaves have mass M_B grams, where $M_B \sim N(50, 4^2)$. Of all loaves produced, 70% are Type A and 30% are Type B.

A loaf is selected at random from all those produced.

(a) Find the probability that its mass is less than 42 g. **[4]**

(b) Given that its mass is less than 42 g, find the probability that it is Type A. **[4]**

7. [Maximum mark: 7] *Difficulty:* 4/5

The machine producing Type A loaves (from Question 6) is adjusted so that the mean mass remains 45 g, but the standard deviation changes to σ g. The machine producing Type B loaves is not adjusted. The probability that the mass of a randomly selected loaf (from these adjusted machines, in the same $70\%/30\%$ proportions) is less than 42 g is now 0.0874.

Find the value of σ. **[7]**

8. [Maximum mark: 8] *Difficulty: 4/5*

The masses of two independent items are given by $X \sim N(25, 4^2)$ kg and $Y \sim N(18, 3^2)$ kg. Find $P(X+Y > 50)$. **[8]**

9. [Maximum mark: 9] *Difficulty: 4/5*

The random variable $X \sim N(\mu, \sigma^2)$ is such that $P(X < 60) = 0.20$ and $P(X < 90) = 0.90$.

Find the value of μ and the value of σ. **[9]**

10. [Maximum mark: 16] *Difficulty: 5/5*

Challenge question. Two shot-put athletes, Marcus and Nadia, are competing. The distances, M metres, thrown by Marcus can be modelled by a normal distribution with mean 8.1 and standard deviation 0.4. The distances, N metres, thrown by Nadia can be modelled by a normal distribution with mean 8.4 and standard deviation 0.35.

In the first round of competition, each athlete has five throws. To qualify for the next round, an athlete must record at least one throw of 8.7 metres or greater in the first round.

(a) Find the probability that Marcus qualifies for the next round. **[4]**

(b) Find the probability that Nadia qualifies for the next round. **[3]**

(c) Find the probability that exactly one of Marcus or Nadia qualifies for the next round. **[4]**

Suppose instead that Nadia is allowed n throws in the first round (still needing at least one throw of 8.7 metres or greater to qualify).

(d) Find the least value of n for which Nadia's probability of qualifying is at least 0.99. **[5]**

Answer Key

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1. (a) $P(X<48)=P\left(Z<\frac{48-55}{7}\right)=P(Z<-1)\approx 0.1587$. (b) $P(X>65)=P\left(Z>\frac{65-55}{7}\right)=P(Z>1.429)\approx 0.0766$.

2. $P(X<a)=0.20 \Rightarrow \frac{a-80}{12}=z_{0.20}\approx -0.8416 \Rightarrow a\approx 69.9$.

3. $P(X>40)=0.25 \Rightarrow P(X<40)=0.75 \Rightarrow \frac{40-\mu}{6}=z_{0.75}\approx 0.6745 \Rightarrow \mu\approx 35.95$.

4. $P(35<X<55)=P(X<55)-P(X<35)\approx 0.8944-0.1056\approx 0.789$.

5. (a) $P(\text{diameter}>53)=P\left(Z>\frac{53-50}{2}\right)=P(Z>1.5)\approx 0.0668$. (b) Using $B\sim \text{Bin}(15,0.0668)$: $P(B=2)=\binom{15}{2}(0.0668)^2(0.9332)^{13}\approx 0.191$.

6. (a) $P(M_A<42)=P\left(Z<\frac{42-45}{3}\right)\approx 0.1587$. $P(M_B<42)=P\left(Z<\frac{42-50}{4}\right)\approx 0.0228$. $P(\text{mass}<42)=0.7(0.1587)+0.3(0.0228)\approx 0.118$. (b) $P(\text{Type } A \mid \text{mass}<42)=\frac{0.7(0.1587)}{0.118}\approx 0.942$.

7. Let $p=P(M_A<42)$ with the new σ. Then $0.7p+0.3(0.0228)=0.0874 \Rightarrow 0.7p=0.0874-0.00683=0.0806 \Rightarrow p\approx 0.1151$. So $\frac{42-45}{\sigma}=z_{0.1151}\approx -1.200 \Rightarrow \sigma\approx 2.50$.

8. Since $X+Y\sim N(43,4^2+3^2)=N(43,25)$: $P(X+Y>50)=P\left(Z>\frac{50-43}{5}\right)=P(Z>1.4)\approx 0.0808$.

9. $\frac{60-\mu}{\sigma}=z_{0.20}\approx -0.8416$ and $\frac{90-\mu}{\sigma}=z_{0.90}\approx 1.2816$. Subtracting: $\frac{30}{\sigma}=1.2816-(-0.8416)=2.1232 \Rightarrow \sigma\approx 14.1$. Then $\mu=60+0.8416(14.1)\approx 71.9$.

10. (a) $P(\text{single throw}\geq 8.7)=P\left(Z\geq \frac{8.7-8.1}{0.4}\right)=P(Z\geq 1.5)\approx 0.0668$. $P(\text{Marcus qualifies})=1-(1-0.0668)^5\approx 0.292$. (b) $P(\text{single throw}\geq 8.7)=P\left(Z\geq \frac{8.7-8.4}{0.35}\right)=P(Z\geq 0.857)\approx 0.1957$. $P(\text{Nadia qualifies})=1-(1-0.1957)^5\approx 0.663$. (c) $P(\text{exactly one qualifies})=0.292(1-0.663)+(1-0.292)(0.663)\approx 0.0984+0.4694\approx 0.568$. (d) Need $1-(1-0.1957)^n\geq 0.99 \Rightarrow (0.8043)^n\leq 0.01 \Rightarrow n\geq \frac{\ln 0.01}{\ln 0.8043}\approx 21.1$. So the least integer value is $n=22$ (check: $n=21$ gives $P\approx 0.9897<0.99$; $n=22$ gives $P\approx 0.9917\geq 0.99$).