The Normal Distribution

Worksheet 1 — Set A

Mathematics: Analysis and Approaches HL

Instructions

- Answer all questions in the spaces provided. Show all working clearly: correct answers without working may not receive full marks.
- Unless told otherwise, give numerical answers exactly or correct to three significant figures.
- Questions are arranged in order of increasing difficulty. Question 10 on each worksheet is a *challenge question*, set beyond typical examination level.
- A calculator may be used unless a question indicates otherwise.

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1. [Maximum mark: 4] *Difficulty:* 1/5

The random variable $X \sim N(40, 6^2)$.

(a) Find $P(X < 35)$. **[2]**

(b) Find $P(X > 50)$. **[2]**

2. [Maximum mark: 5] *Difficulty:* 1/5

The random variable $X \sim N(70, 10^2)$. Find the value of a such that $P(X > a) = 0.10$. **[5]**

3. [Maximum mark: 5] *Difficulty:* 2/5

The random variable $X \sim N(\mu, 4^2)$. Given that $P(X < 25) = 0.15$, find the value of μ. **[5]**

4. [Maximum mark: 6] *Difficulty:* 2/5

The random variable $X \sim N(30, 5^2)$. Find $P(25 < X < 38)$. **[6]**

5. [Maximum mark: 7] *Difficulty:* 3/5

The masses of eggs produced on a farm are normally distributed with mean 65 g and standard deviation 3 g. An egg is graded "large" if its mass exceeds 70 g.

(a) Find the probability that a randomly selected egg is graded large. **[2]**

(b) In a random sample of 12 eggs, find the probability that exactly 3 are graded large. **[5]**

6. [Maximum mark: 8] *Difficulty:* 3/5

A factory produces two types of component. Type A components have mass M_A grams, where $M_A \sim N(50, 4^2)$. Type B components have mass M_B grams, where $M_B \sim N(55, 5^2)$. Of all components produced, 65% are Type A and 35% are Type B.

A component is selected at random from all those produced.

(a) Find the probability that its mass is less than 48 g. **[4]**

(b) Given that its mass is less than 48 g, find the probability that it is Type A. **[4]**

7. [Maximum mark: 7] *Difficulty:* 4/5

The machine producing Type A components (from Question 6) is adjusted so that the mean mass remains 50 g, but the standard deviation changes to σ g. The machine producing Type B components is not adjusted. The probability that the mass of a randomly selected component (from these adjusted machines, in the same 65%/35% proportions) is less than 48 g is now 0.192.

Find the value of σ. **[7]**

8. [Maximum mark: 8] *Difficulty:* 4/5

The masses of two independent items are given by $X \sim N(20, 3^2)$ kg and $Y \sim N(15, 2^2)$ kg. Find $P(X + Y > 40)$. **[8]**

9. [Maximum mark: 9] *Difficulty:* 4/5

The random variable $X \sim N(\mu, \sigma^2)$ is such that $P(X < 50) = 0.30$ and $P(X < 70) = 0.80$.

Find the value of μ and the value of σ. **[9]**

10. [Maximum mark: 16] *Difficulty:* 5/5

Challenge question. Two long-jump athletes, Kofi and Lena, are competing. The distances, K metres, jumped by Kofi can be modelled by a normal distribution with mean 6.2 and standard deviation 0.3. The distances, L metres, jumped by Lena can be modelled by a normal distribution with mean 6.4 and standard deviation 0.25.

In the first round of competition, each athlete has four jumps. To qualify for the next round, an athlete must record at least one jump of 6.5 metres or greater in the first round.

(a) Find the probability that Kofi qualifies for the next round. **[4]**

(b) Find the probability that Lena qualifies for the next round. **[3]**

(c) Find the probability that exactly one of Kofi or Lena qualifies for the next round. **[4]**

Suppose instead that Lena is allowed n jumps in the first round (still needing at least one jump of 6.5 metres or greater to qualify).

(d) Find the least value of n for which Lena's probability of qualifying is at least 0.995. **[5]**

Answer Key

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1. (a) $P(X<35)=P\left(Z<\frac{35-40}{6}\right)=P(Z<-0.833)\approx 0.202.$ (b) $P(X>50)=P\left(Z>\frac{50-40}{6}\right)=P(Z>1.667)\approx 0.0478.$

2. $P(X>a)=0.10\Rightarrow P(X<a)=0.90\Rightarrow \frac{a-70}{10}=z_{0.90}\approx 1.2816\Rightarrow a\approx 82.8.$

3. $P(X<25)=0.15\Rightarrow \frac{25-\mu}{4}=z_{0.15}\approx -1.0364\Rightarrow \mu=25+4(1.0364)\approx 29.1.$

4. $P(25<X<38)=P(X<38)-P(X<25)\approx 0.9452-0.1587\approx 0.787.$

5. (a) $P(\text{mass}>70)=P\left(Z>\frac{70-65}{3}\right)=P(Z>1.667)\approx 0.0478.$ (b) Using $B\sim \text{Bin}(12,0.0478)$: $P(B=3)=\binom{12}{3}(0.0478)^3(0.9522)^9\approx 0.0155.$

6. (a) $P(M_A<48)=P\left(Z<\frac{48-50}{4}\right)\approx 0.3085.$ $P(M_B<48)=P\left(Z<\frac{48-55}{5}\right)\approx 0.0808.$ $P(\text{mass}<48)=0.65(0.3085)+0.35(0.0808)\approx 0.229.$ (b) $P(\text{Type } A\mid \text{mass}<48)=\frac{0.65(0.3085)}{0.229}\approx 0.876.$

7. Let $p=P(M_A<48)$ with the new σ. Then $0.65p+0.35(0.0808)=0.192\Rightarrow 0.65p=0.192-0.0283=0.1637\Rightarrow p\approx 0.2519$. So $\frac{48-50}{\sigma}=z_{0.2519}\approx -0.667\Rightarrow \sigma\approx 3.00.$

8. Since $X+Y\sim N(35,3^2+2^2)=N(35,13)$: $P(X+Y>40)=P\left(Z>\frac{40-35}{\sqrt{13}}\right)=P(Z>1.387)\approx 0.0828.$

9. $\frac{50-\mu}{\sigma}=z_{0.30}\approx -0.5244$ and $\frac{70-\mu}{\sigma}=z_{0.80}\approx 0.8416$. Subtracting: $\frac{20}{\sigma}=0.8416-(-0.5244)=1.3660\Rightarrow \sigma\approx 14.6$. Then $\mu=50+0.5244(14.6)\approx 57.7.$

10. (a) $P(\text{single jump}\geq 6.5)=P\left(Z\geq \frac{6.5-6.2}{0.3}\right)=P(Z\geq 1)\approx 0.1587.$ $P(\text{Kofi qualifies})=1-(1-0.1587)^4\approx 0.499.$ (b) $P(\text{single jump}\geq 6.5)=P\left(Z\geq \frac{6.5-6.4}{0.25}\right)=P(Z\geq 0.4)\approx 0.3446.$ $P(\text{Lena qualifies})=1-(1-0.3446)^4\approx 0.815.$ (c) $P(\text{exactly one qualifies})=0.499(1-0.815)+(1-0.499)(0.815)\approx 0.0922+0.4083\approx 0.501.$ (d) Need $1-(1-0.3446)^n\geq 0.995\Rightarrow (0.6554)^n\leq 0.005\Rightarrow n\geq \frac{\ln 0.005}{\ln 0.6554}\approx 12.54$. So the least integer value is $n=13$ (check: $n=12$ gives $P\approx 0.9937<0.995$; $n=13$ gives $P\approx 0.9959\geq 0.995$).