

# Kinematics

## Worksheet 3 — Set C

Mathematics: Analysis and Approaches HL

### Instructions

- Answer all questions in the spaces provided. Show all working clearly: correct answers without working may not receive full marks.
- Unless told otherwise, give numerical answers exactly or correct to three significant figures.
- Questions are arranged in order of increasing difficulty. Question 10 on each worksheet is a *challenge question*, set beyond typical examination level.
- A calculator may be used unless a question indicates otherwise.

#### 1. [Maximum mark: 4]

Difficulty: 1/5

A particle moves in a straight line such that its velocity,  $v \text{ ms}^{-1}$ , at time  $t$  seconds is given by  $v(t) = 5t^2 - 15t + 10$ , where  $t \geq 0$ .

Find the times when the particle is momentarily at rest.

[4]

#### 2. [Maximum mark: 6]

Difficulty: 1/5

The displacement of a particle,  $s$  metres, at time  $t$  seconds is given by  $s(t) = t^4 + t^3 - 4t^2$ .

(a) Find expressions for the velocity  $v(t)$  and acceleration  $a(t)$ . [3]

(b) Find the velocity and acceleration of the particle when  $t = 1$ . [3]

#### 3. [Maximum mark: 6]

Difficulty: 2/5

A particle moves in a straight line with velocity  $v(t) = 4 \sin(3t) \text{ ms}^{-1}$ , for  $0 \leq t \leq \frac{\pi}{3}$ .

Find the total distance travelled by the particle.

[6]

#### 4. [Maximum mark: 7]

Difficulty: 2/5

A particle moves in a straight line with velocity  $v(t) = t^2 e^{-0.5t} \text{ ms}^{-1}$ , for  $0 \leq t \leq 8$ .

Find the acceleration of the particle when its speed is greatest.

[7]

#### 5. [Maximum mark: 8]

Difficulty: 3/5

A particle moves in a straight line such that its velocity,  $v \text{ ms}^{-1}$ , at time  $t$  seconds is given by

$$v = \frac{(t^2 - 4) \sin t}{5}, \quad 0 \leq t \leq 5.$$

(a) Determine the times when the particle changes its direction of motion. [3]

(b) Find the acceleration of the particle when its speed is greatest. [5]

#### 6. [Maximum mark: 9]

Difficulty: 3/5

A particle moves in a straight line so that its velocity,  $v \text{ ms}^{-1}$ , after  $t$  seconds is given by

$$v(t) = 2 \cos t - e^{0.4 \sin t}, \quad 0 \leq t \leq 6.$$

- (a) Find the values of  $t$  when the particle is at rest. [3]
- (b) Find the acceleration of the particle when it changes direction. [3]
- (c) Find the total distance travelled by the particle. [3]

**7. [Maximum mark: 6]***Difficulty: 4/5*

A particle moves in a straight line with velocity  $v(t) = 3t - 9 \text{ ms}^{-1}$ , for  $0 \leq t \leq 5$ .

- (a) Find the average velocity of the particle over this interval. [3]
- (b) Find the average speed of the particle over this interval. [3]

**8. [Maximum mark: 8]***Difficulty: 4/5*

The acceleration of a particle moving in a straight line is given by  $a(t) = 2t - 6 \text{ ms}^{-2}$ . When  $t = 0$ , the particle has velocity  $5 \text{ ms}^{-1}$  and displacement  $-2 \text{ m}$ .

- (a) Find an expression for the velocity  $v(t)$ . [3]
- (b) Find an expression for the displacement  $s(t)$ , and hence find the displacement and velocity when  $t = 3$ . [5]

**9. [Maximum mark: 8]***Difficulty: 4/5*

A particle moves in a straight line with velocity  $v(t) = 2t^2 - 14t + 12 \text{ ms}^{-1}$ . Initially the particle is at the origin.

Find the times, other than  $t = 0$ , when the particle returns to the origin. [8]

**10. [Maximum mark: 16]***Difficulty: 5/5*

**Challenge question.** A particle moves in a straight line such that its velocity,  $v \text{ ms}^{-1}$ , at time  $t$  seconds is given by

$$v(t) = (t + 1)e^{-0.25t} \sin t, \quad 0 \leq t \leq 6.$$

- (a) Find the value of  $t$ , other than  $t = 0$ , when the particle changes its direction of motion. [4]
- (b) Find the times when the particle's acceleration is  $-0.5 \text{ ms}^{-2}$ . [5]
- (c) Find the particle's acceleration when its speed is greatest. [4]
- (d) Find the total distance travelled by the particle. [3]

## Answer Key

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1.  $5t^2 - 15t + 10 = 0 \Rightarrow 5(t-1)(t-2) = 0 \Rightarrow t = 1$  or  $t = 2$ .
2. (a)  $v(t) = 4t^3 + 3t^2 - 8t$ ;  $a(t) = 12t^2 + 6t - 8$ . (b)  $v(1) = 4 + 3 - 8 = -1 \text{ ms}^{-1}$ ;  $a(1) = 12 + 6 - 8 = 10 \text{ ms}^{-2}$ .
3. Since  $v(t) = 4\sin(3t) \geq 0$  throughout  $[0, \frac{\pi}{3}]$  (as  $3t$  ranges over  $[0, \pi]$ ): distance  $= \int_0^{\pi/3} 4\sin(3t) dt = \frac{8}{3} \text{ m}$ .
4. Maximum speed occurs where  $a(t) = v'(t) = 0$ :  $v'(t) = 2te^{-0.5t} - 0.5t^2e^{-0.5t} = te^{-0.5t}(2 - 0.5t) = 0 \Rightarrow t = 4$  (rejecting  $t = 0$ ). At  $t = 4$ ,  $a(t) = 0 \text{ ms}^{-2}$ .
5. (a) Solving  $v = 0$  numerically (GDC) on  $(0, 5]$ :  $t = 2$  and  $t = \pi \approx 3.14$ . (b) Using GDC to maximise  $|v(t)|$ : greatest speed occurs at  $t \approx 0.990$ , where  $a(t) \approx 0 \text{ ms}^{-2}$ .
6. (a) Solving  $v(t) = 0$  numerically (GDC):  $t \approx 0.833$  and  $t \approx 5.06$ . (b) Evaluating  $a(t) = v'(t)$  at these times gives  $a \approx -1.84 \text{ ms}^{-2}$  at  $t \approx 0.833$  and  $a \approx 1.78 \text{ ms}^{-2}$  at  $t \approx 5.06$ . (c) Total distance  $= \int_0^6 |v(t)| dt \approx 9.03 \text{ m}$ .
7. (a)  $s(t) = 1.5t^2 - 9t$ .  $s(5) = -7.5$ ,  $s(0) = 0$ . Average velocity  $= \frac{-7.5 - 0}{5} = -1.5 \text{ ms}^{-1}$ . (b) Average speed  $= \frac{1}{5} \int_0^5 |3t - 9| dt = \frac{39}{10} = 3.9 \text{ ms}^{-1}$ .
8. (a)  $v(t) = \int (2t - 6) dt = t^2 - 6t + 5$  (using  $v(0) = 5$ ). (b)  $s(t) = \int (t^2 - 6t + 5) dt = \frac{1}{3}t^3 - 3t^2 + 5t - 2$  (using  $s(0) = -2$ ).  $s(3) = 9 - 27 + 15 - 2 = -5 \text{ m}$ ;  $v(3) = 9 - 18 + 5 = -4 \text{ ms}^{-1}$ .
9.  $s(t) = \int (2t^2 - 14t + 12) dt = \frac{2}{3}t^3 - 7t^2 + 12t$  (using  $s(0) = 0$ ). Solving  $s(t) = 0$  for  $t > 0$  numerically (GDC):  $t \approx 2.16$  or  $t \approx 8.34$ .
10. (a) Solving  $v(t) = 0$  numerically (GDC) for  $t > 0$ :  $t = \pi \approx 3.14$ . (b) Solving  $a(t) = -0.5$  numerically (GDC):  $t \approx 1.94$  and  $t \approx 4.37$ . (c) Maximising  $|v(t)|$  numerically (GDC): greatest speed occurs at  $t \approx 1.69$ , where  $a(t) \approx 0 \text{ ms}^{-2}$ . (d) Total distance  $= \int_0^6 |v(t)| dt \approx 6.80 \text{ m}$ .