# Kinematics

Worksheet 2 — Set B

*Mathematics: Analysis and Approaches HL*

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**Instructions**

- Answer all questions in the spaces provided. Show all working clearly: correct answers without working may not receive full marks.
- Unless told otherwise, give numerical answers exactly or correct to three significant figures.
- Questions are arranged in order of increasing difficulty. Question 10 on each worksheet is a *challenge question*, set beyond typical examination level.
- A calculator may be used unless a question indicates otherwise.

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**1. [Maximum mark: 4]** *Difficulty:* 1/5

A particle moves in a straight line such that its velocity, $v$ ms$^{-1}$, at time $t$ seconds is given by $v(t) = 3t^2 - 14t + 8$, where $t \geq 0$.

Find the times when the particle is momentarily at rest. **[4]**

**2. [Maximum mark: 6]** *Difficulty:* 1/5

The displacement of a particle, $s$ metres, at time $t$ seconds is given by $s(t) = t^4 - 2t^3 - 3t^2$.

(a) Find expressions for the velocity $v(t)$ and acceleration $a(t)$. **[3]**

(b) Find the velocity and acceleration of the particle when $t = 1$. **[3]**

**3. [Maximum mark: 6]** *Difficulty:* 2/5

A particle moves in a straight line with velocity $v(t) = 6\cos(3t)$ ms$^{-1}$, for $0 \leq t \leq \dfrac{\pi}{2}$.

Find the total distance travelled by the particle. **[6]**

**4. [Maximum mark: 7]** *Difficulty:* 2/5

A particle moves in a straight line with velocity $v(t) = t^3 e^{-t}$ ms$^{-1}$, for $0 \leq t \leq 6$.

Find the acceleration of the particle when its speed is greatest. **[7]**

**5. [Maximum mark: 8]** *Difficulty:* 3/5

A particle moves in a straight line such that its velocity, $v$ ms$^{-1}$, at time $t$ seconds is given by

$$v = \frac{(t^2 - 5)\cos t}{4}, \qquad 0 \leq t \leq 5.$$

(a) Determine the times when the particle changes its direction of motion. **[3]**

(b) Find the acceleration of the particle when its speed is greatest. **[5]**

**6. [Maximum mark: 9]** *Difficulty:* 3/5

A particle moves in a straight line so that its velocity, $v$ ms$^{-1}$, after $t$ seconds is given by

$$v(t) = 3\sin t - e^{0.5\cos t}, \qquad 0 \leq t \leq 6.$$

(a) Find the values of $t$ when the particle is at rest. **[3]**

(b) Find the acceleration of the particle when it changes direction. **[3]**

(c) Find the total distance travelled by the particle. **[3]**

**7. [Maximum mark: 6]** *Difficulty:* 4/5

A particle moves in a straight line with velocity $v(t) = 6t - 12$ ms$^{-1}$, for $0 \le t \le 5$.

(a) Find the average velocity of the particle over this interval. **[3]**

(b) Find the average speed of the particle over this interval. **[3]**

**8. [Maximum mark: 8]** *Difficulty:* 4/5

The acceleration of a particle moving in a straight line is given by $a(t) = 4t + 2$ ms$^{-2}$. When $t = 0$, the particle has velocity $-1$ ms$^{-1}$ and displacement 3 m.

(a) Find an expression for the velocity $v(t)$. **[3]**

(b) Find an expression for the displacement $s(t)$, and hence find the displacement and velocity when $t = 3$. **[5]**

**9. [Maximum mark: 8]** *Difficulty:* 4/5

A particle moves in a straight line with velocity $v(t) = 2t^2 - 12t + 10$ ms$^{-1}$. Initially the particle is at the origin.

Find the times, other than $t = 0$, when the particle returns to the origin. **[8]**

**10. [Maximum mark: 16]** *Difficulty:* 5/5

**Challenge question.** A particle moves in a straight line such that its velocity, $v$ ms$^{-1}$, at time $t$ seconds is given by

$$v(t) = (t-1)e^{-0.3t}\cos t, \qquad 0 \le t \le 6.$$

(a) Find the values of $t$, other than $t = 1$, when the particle changes its direction of motion. **[4]**

(b) Find the times when the particle's acceleration is 0.3 ms$^{-2}$. **[5]**

(c) Find the particle's acceleration when its speed is greatest. **[4]**

(d) Find the total distance travelled by the particle. **[3]**

# Answer Key

**1.** $3t^2 - 14t + 8 = 0 \Rightarrow (3t-2)(t-4) = 0 \Rightarrow t = \dfrac{2}{3}$ or $t = 4$.

**2.** (a) $v(t) = 4t^3 - 6t^2 - 6t$; $a(t) = 12t^2 - 12t - 6$. (b) $v(1) = 4 - 6 - 6 = -8$ ms$^{-1}$; $a(1) = 12 - 12 - 6 = -6$ ms$^{-2}$.

**3.** Since $v(t) = 6\cos(3t)$ is non-negative on $[0, \frac{\pi}{6}]$ and non-positive on $[\frac{\pi}{6}, \frac{\pi}{2}]$: distance $= \displaystyle\int_0^{\pi/2} |6\cos(3t)|\,dt =$ 6 m.

**4.** Maximum speed occurs where $a(t) = v'(t) = 0$: $v'(t) = 3t^2e^{-t} - t^3e^{-t} = t^2e^{-t}(3-t) = 0 \Rightarrow t = 3$ (rejecting $t = 0$). At $t = 3$, $a(t) = 0$ ms$^{-2}$.

**5.** (a) Solving $v = 0$ numerically (GDC) on $(0,5]$: $t \approx 1.57$ $(= \frac{\pi}{2})$, $t = \sqrt{5} \approx 2.24$, and $t \approx 4.71$ $(= \frac{3\pi}{2})$. (b) Using GDC to maximise $|v(t)|$: greatest speed occurs at $t \approx 3.82$, where $a(t) \approx 0$ ms$^{-2}$.

**6.** (a) Solving $v(t) = 0$ numerically (GDC): $t \approx 0.538$ and $t \approx 2.94$. (b) Evaluating $a(t) = v'(t)$ at these times gives $a \approx 2.97$ ms$^{-2}$ at $t \approx 0.538$ and $a \approx -2.87$ ms$^{-2}$ at $t \approx 2.94$. (c) Total distance $= \displaystyle\int_0^6 |v(t)|\,dt \approx$ 12.4 m.

**7.** (a) $s(t) = 3t^2 - 12t$. $s(5) = 15$, $s(0) = 0$. Average velocity $= \dfrac{15-0}{5} = 3$ ms$^{-1}$. (b) Average speed $= \dfrac{1}{5}\displaystyle\int_0^5 |6t - 12|\,dt = \dfrac{39}{5} = 7.8$ ms$^{-1}$.

**8.** (a) $v(t) = \displaystyle\int (4t+2)\,dt = 2t^2 + 2t - 1$ (using $v(0) = -1$). (b) $s(t) = \displaystyle\int (2t^2 + 2t - 1)\,dt = \frac{2}{3}t^3 + t^2 - t + 3$ (using $s(0) = 3$). $s(3) = 18 + 9 - 3 + 3 = 27$ m; $v(3) = 18 + 6 - 1 = 23$ ms$^{-1}$.

**9.** $s(t) = \displaystyle\int (2t^2 - 12t + 10)\,dt = \frac{2}{3}t^3 - 6t^2 + 10t$ (using $s(0) = 0$). Solving $s(t) = 0$ for $t > 0$: $t^2 - 9t + 15 = 0 \Rightarrow t = \dfrac{9 \pm \sqrt{21}}{2}$, giving $t \approx 2.21$ or $t \approx 6.79$.

**10.** (a) Solving $v(t) = 0$ numerically (GDC) for $t \neq 1$: $t \approx 1.57$ $(= \frac{\pi}{2})$ and $t \approx 4.71$ $(= \frac{3\pi}{2})$. (b) Solving $a(t) = 0.3$ numerically (GDC): $t \approx 1.07$, $t \approx 3.58$, and $t \approx 5.83$. (c) Maximising $|v(t)|$ numerically (GDC): greatest speed occurs at $t \approx 3.28$, where $a(t) \approx 0$ ms$^{-2}$. (d) Total distance $= \displaystyle\int_0^6 |v(t)|\,dt \approx 2.65$ m.