Kinematics

Worksheet 1 — Set A

Mathematics: Analysis and Approaches HL

Instructions

- Answer all questions in the spaces provided. Show all working clearly: correct answers without working may not receive full marks.
- Unless told otherwise, give numerical answers exactly or correct to three significant figures.
- Questions are arranged in order of increasing difficulty. Question 10 on each worksheet is a *challenge question*, set beyond typical examination level.
- A calculator may be used unless a question indicates otherwise.

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1. [Maximum mark: 4] *Difficulty:* 1/5

A particle moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by $v(t) = 3t^2 - 12t + 9$, where $t \geq 0$.

Find the times when the particle is momentarily at rest. **[4]**

2. [Maximum mark: 6] *Difficulty:* 1/5

The displacement of a particle, s metres, at time t seconds is given by $s(t) = t^4 - 4t^3 + 2t$.

(a) Find expressions for the velocity $v(t)$ and acceleration $a(t)$. **[3]**

(b) Find the velocity and acceleration of the particle when $t = 1$. **[3]**

3. [Maximum mark: 6] *Difficulty:* 2/5

A particle moves in a straight line with velocity $v(t) = 5\sin(2t)$ ms^{-1}, for $0 \leq t \leq \pi$.

Find the total distance travelled by the particle. **[6]**

4. [Maximum mark: 7] *Difficulty:* 2/5

A particle moves in a straight line with velocity $v(t) = t^2 e^{-t}$ ms^{-1}, for $0 \leq t \leq 5$.

Find the acceleration of the particle when its speed is greatest. **[7]**

5. [Maximum mark: 8] *Difficulty:* 3/5

A particle moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by

$$v = \frac{(t^2 - 2)\sin t}{3}, \qquad 0 \leq t \leq 4.$$

(a) Determine the times when the particle changes its direction of motion. **[3]**

(b) Find the acceleration of the particle when its speed is greatest. **[5]**

6. [Maximum mark: 9] *Difficulty:* 3/5

A particle moves in a straight line so that its velocity, v ms^{-1}, after t seconds is given by

$$v(t) = e^{\cos t} - 3\cos t, \qquad 0 \leq t \leq 5.$$

(a) Find the value of t when the particle is at rest. **[3]**

(b) Find the acceleration of the particle when it changes direction. **[3]**

(c) Find the total distance travelled by the particle. **[3]**

7. [Maximum mark: 6] *Difficulty:* 4/5

A particle moves in a straight line with velocity $v(t) = 4t - 8$ ms^{-1}, for $0 \leq t \leq 4$.

(a) Find the average velocity of the particle over this interval. **[3]**

(b) Find the average speed of the particle over this interval. **[3]**

8. [Maximum mark: 8] *Difficulty:* 4/5

The acceleration of a particle moving in a straight line is given by $a(t) = 6t - 4$ ms^{-2}. When $t = 0$, the particle has velocity 2 ms^{-1} and displacement 0 m.

(a) Find an expression for the velocity $v(t)$. **[3]**

(b) Find an expression for the displacement $s(t)$, and hence find the displacement and velocity when $t = 3$. **[5]**

9. [Maximum mark: 8] *Difficulty:* 4/5

A particle moves in a straight line with velocity $v(t) = 3t^2 - 15t + 12$ ms^{-1}. Initially the particle is at the origin.

Find the times, other than $t = 0$, when the particle returns to the origin. **[8]**

10. [Maximum mark: 16] *Difficulty:* 5/5

Challenge question. A particle moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by

$$v(t) = te^{-0.5t}\sin t, \qquad 0 \leq t \leq 6.$$

(a) Find the value of t, other than $t = 0$, when the particle changes its direction of motion. **[4]**

(b) Find the times when the particle's acceleration is -0.4 ms^{-2}. **[5]**

(c) Find the particle's acceleration when its speed is greatest. **[4]**

(d) Find the total distance travelled by the particle. **[3]**

Answer Key

1. $3t^2 - 12t + 9 = 0 \Rightarrow 3(t-1)(t-3) = 0 \Rightarrow t = 1$ or $t = 3$.

2. (a) $v(t) = 4t^3 - 12t^2 + 2$; $a(t) = 12t^2 - 24t$. (b) $v(1) = 4 - 12 + 2 = -6$ ms^{-1}; $a(1) = 12 - 24 = -12$ ms^{-2}.

3. Since $v(t) = 5\sin(2t)$ is non-negative on $[0, \frac{\pi}{2}]$ and non-positive on $[\frac{\pi}{2}, \pi]$: distance $= \int_0^\pi |5\sin(2t)|\,dt = 10$ m.

4. Maximum speed occurs where $a(t) = v'(t) = 0$: $v'(t) = 2te^{-t} - t^2e^{-t} = te^{-t}(2-t) = 0 \Rightarrow t = 2$ (rejecting $t = 0$, a minimum). At $t = 2$, $a(t) = 0$ ms^{-2} (since this is precisely where the velocity is stationary).

5. (a) Solving $v = 0$ numerically (GDC) on $(0, 4]$: $t = \sqrt{2} \approx 1.41$ and $t = \pi \approx 3.14$. (b) Using GDC to maximise $|v(t)|$: greatest speed occurs at $t \approx 2.45$, where $a(t) = v'(t) \approx 0$ ms^{-2}.

6. (a) Solving $v(t) = 0$ numerically (GDC): $t \approx 0.903$. (b) $a(t) = v'(t)$; evaluating at $t \approx 0.903$ gives $a \approx 0.898$ ms^{-2}. (c) Since $v(t) < 0$ for $0 \le t < 0.903$ and $v(t) > 0$ for $0.903 < t \le 5$, total distance $= \int_0^5 |v(t)|\,dt \approx 8.43$ m.

7. (a) Average velocity $= \dfrac{s(4) - s(0)}{4}$. Since $s(t) = 2t^2 - 8t$, $s(4) = 0$, $s(0) = 0$, so average velocity $= 0$ ms^{-1}. (b) Average speed $= \dfrac{1}{4}\int_0^4 |4t - 8|\,dt = \dfrac{16}{4} = 4$ ms^{-1}.

8. (a) $v(t) = \int (6t - 4)\,dt = 3t^2 - 4t + 2$ (using $v(0) = 2$). (b) $s(t) = \int (3t^2 - 4t + 2)\,dt = t^3 - 2t^2 + 2t$ (using $s(0) = 0$). $s(3) = 27 - 18 + 6 = 15$ m; $v(3) = 27 - 12 + 2 = 17$ ms^{-1}.

9. $s(t) = \int (3t^2 - 15t + 12)\,dt = t^3 - \frac{15}{2}t^2 + 12t$ (using $s(0) = 0$). Solving $s(t) = 0$ for $t > 0$: $t^2 - \frac{15}{2}t + 12 = 0 \Rightarrow 2t^2 - 15t + 24 = 0 \Rightarrow t = \dfrac{15 \pm \sqrt{33}}{4}$, giving $t \approx 2.31$ or $t \approx 5.19$.

10. (a) Solving $v(t) = 0$ numerically (GDC) for $t > 0$: $t = \pi \approx 3.14$. (b) Solving $a(t) = -0.4$ numerically (GDC): $t \approx 2.12$ and $t \approx 3.74$. (c) Maximising $|v(t)|$ numerically (GDC): greatest speed occurs at $t \approx 1.67$, where $a(t) \approx 0$ ms^{-2}. (d) Total distance $= \int_0^6 |v(t)|\,dt \approx 2.18$ m.