Geometry

Worksheet 1 — Set A

Mathematics: Analysis and Approaches HL

Instructions

- Answer all questions in the spaces provided. Show all working clearly: correct answers without working may not receive full marks.
- Unless told otherwise, give numerical answers exactly or correct to three significant figures.
- Questions are arranged in order of increasing difficulty. Question 10 on each worksheet is a *challenge question*, set beyond typical examination level.
- A calculator may be used unless a question indicates otherwise.

---

1. [Maximum mark: 3] *Difficulty:* 1/5

(a) Convert $150°$ to radians, giving your answer as a multiple of π. **[1]**

(b) Convert $\dfrac{5\pi}{4}$ radians to degrees. **[2]**

2. [Maximum mark: 4] *Difficulty:* 1/5

A circle has radius 8 cm. Find

(a) the length of an arc that subtends an angle of 1.2 radians at the centre; **[2]**

(b) the area of the corresponding sector. **[2]**

3. [Maximum mark: 5] *Difficulty:* 2/5

A circle has centre O and radius 6 m. Points A and B lie on the circle such that $A\hat{O}B = 1.4$ radians.

(a) Find the length of the chord $[AB]$. **[3]**

(b) Find the area of the minor sector OAB. **[2]**

4. [Maximum mark: 6] *Difficulty:* 2/5

Triangle ABC has $AB = 7$ cm, $BC = 9$ cm, and $A\hat{B}C = 50°$.

(a) Find AC. **[3]**

(b) Find the area of triangle ABC. **[3]**

5. [Maximum mark: 7] *Difficulty:* 3/5

Consider a circle with diameter AB, where A has coordinates $(2,0,0)$ and B has coordinates $(-2,0,4)$.

(a) **(i)** Find the coordinates of the centre of the circle. **[2]**

(ii) Find the radius of the circle. **[2]**

The circle forms the base of a right cone whose vertex V has coordinates $(0,5,2)$.

(b) Find the exact volume of the cone. **[3]**

6. [Maximum mark: 6] *Difficulty:* 3/5

A circle has centre O and radius 4 cm. Points C and D lie on the circle such that $C\hat{O}D = \theta$, where $0 < \theta < \pi$.

(a) Find an expression, in terms of θ, for the area of the minor segment cut off by chord $[CD]$. **[3]**

(b) Hence find the area of this segment when $\theta = 1.5$ radians. **[3]**

7. [Maximum mark: 8] *Difficulty:* 4/5

A company designs a logo by removing two equal segments from a rectangle measuring 6 cm by 5 cm. The points A and B lie on a circle, with centre O and radius 2.5 cm, such that $A\hat{O}B = \theta$, where $0 < \theta < \pi$.

(a) Find the area of one of the shaded segments in terms of θ. **[3]**

(b) Given that the area of the logo is 22.5 cm^2, find the value of θ. **[5]**

8. [Maximum mark: 7] *Difficulty:* 4/5

Consider a quadrilateral $ABCD$ such that $AB = 3$, $BC = 5$, $CD = 7$ and $DA = 9$. Let $\alpha = A\hat{B}C$ and $\beta = A\hat{D}C$.

(a) **(i)** Find AC^2 in terms of α. **[2]**

(ii) Find AC^2 in terms of β. **[2]**

(iii) Hence, or otherwise, find an expression for α in terms of β. **[3]**

9. [Maximum mark: 8] *Difficulty:* 4/5

Using the quadrilateral $ABCD$ from Question 8, find the maximum area of $ABCD$. **[8]**

10. [Maximum mark: 13] *Difficulty:* 5/5

Challenge question. A sector of a circle has radius r cm and angle θ radians, where $0 < \theta < 2\pi$. The perimeter of the sector (the two straight edges together with the arc) is fixed at 20 cm.

(a) Show that $\theta = \dfrac{20}{r} - 2$. **[3]**

(b) Show that the area of the sector is given by $A(r) = 10r - r^2$. **[3]**

(c) Find the value of r that maximises $A(r)$, and find this maximum area. **[4]**

(d) Find the corresponding value of θ at this maximum, and confirm that it satisfies $0 < \theta < 2\pi$. **[3]**

Answer Key

1. (a) $150^\circ = \frac{5\pi}{6}$. (b) $\frac{5\pi}{4} = 225^\circ$.

2. (a) Arc length $= r\theta = 8(1.2) = 9.6$ cm. (b) Sector area $= \frac{1}{2}r^2\theta = \frac{1}{2}(64)(1.2) = 38.4$ cm^2.

3. (a) $AB^2 = 6^2 + 6^2 - 2(6)(6)\cos(1.4) = 72(1-\cos 1.4)$, so $AB \approx 7.73$ m. (b) Sector area $= \frac{1}{2}(36)(1.4) = 25.2$ m^2.

4. (a) $AC^2 = 7^2 + 9^2 - 2(7)(9)\cos 50^\circ \approx 49.01$, so $AC \approx 7.00$ cm. (b) Area $= \frac{1}{2}(7)(9)\sin 50^\circ \approx 24.1$ cm^2.

5. (a)(i) Centre $=$ midpoint of $AB = (0,0,2)$. (ii) Radius $= \frac{1}{2}|AB| = \frac{1}{2}\sqrt{4^2+0^2+4^2} = \frac{1}{2}\sqrt{32} = 2\sqrt{2}$. (b) Height $=$ distance from V to centre $= \sqrt{0^2+5^2+0^2} = 5$. Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8)(5) = \frac{40\pi}{3}$.

6. (a) Segment area $= \frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{2}(16)(\theta - \sin\theta) = 8(\theta - \sin\theta)$. (b) At $\theta = 1.5$: $8(1.5 - \sin 1.5) \approx 4.02$ cm^2.

7. (a) Segment area $= \frac{1}{2}(2.5)^2(\theta - \sin\theta) = 3.125(\theta - \sin\theta)$. (b) Logo area $=$ rectangle area $-2\times$ segment area: $30 - 6.25(\theta - \sin\theta) = 22.5 \Rightarrow \theta - \sin\theta = 1.2$. Solving numerically (GDC): $\theta \approx 2.08$ radians.

8. (a)(i) $AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\alpha = 9 + 25 - 30\cos\alpha = 34 - 30\cos\alpha$. (ii) $AC^2 = DA^2 + CD^2 - 2(DA)(CD)\cos\beta = 81 + 49 - 126\cos\beta = 130 - 126\cos\beta$. (iii) Equating: $34 - 30\cos\alpha = 130 - 126\cos\beta \Rightarrow \cos\alpha = \frac{126\cos\beta - 96}{30} = \frac{21\cos\beta - 16}{5}$, so $\alpha = \arccos\left(\frac{21\cos\beta - 16}{5}\right)$.

9. Area $= \frac{1}{2}(3)(5)\sin\alpha + \frac{1}{2}(7)(9)\sin\beta = 7.5\sin\alpha + 31.5\sin\beta$, subject to the relationship in Question 8(a)(iii). Maximising numerically (GDC) gives a maximum area of approximately 30.7 (square units), occurring at $\alpha \approx 2.23$, $\beta \approx 0.908$ radians.

10. (a) Perimeter $= 2r + r\theta = 20 \Rightarrow r\theta = 20 - 2r \Rightarrow \theta = \frac{20-2r}{r} = \frac{20}{r} - 2$. (b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}r^2\left(\frac{20}{r} - 2\right) = 10r - r^2$. (c) $A'(r) = 10 - 2r = 0 \Rightarrow r = 5$. Since $A''(r) = -2 < 0$, this is a maximum: $A(5) = 10(5) - 5^2 = 25$ cm^2. (d) $\theta = \frac{20}{5} - 2 = 2$ radians, and indeed $0 < 2 < 2\pi$. ✓