Exponents and Logarithms

Worksheet 3 — Set C

Mathematics: Analysis and Approaches HL

Instructions

- Answer all questions in the spaces provided. Show all working clearly: correct answers without working may not receive full marks.
- Unless told otherwise, give numerical answers exactly or correct to three significant figures.
- Questions are arranged in order of increasing difficulty. Question 10 on each worksheet is a *challenge question*, set beyond typical examination level.
- A calculator may be used unless a question indicates otherwise.

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1. [Maximum mark: 4] *Difficulty:* 1/5

Simplify, giving your answer as a single power.

(a) $7^0 \times 7^3$ **[2]**

(b) $\dfrac{(5^2)^3}{5^4}$ **[2]**

2. [Maximum mark: 5] *Difficulty:* 1/5

Without using a calculator, evaluate each of the following.

(a) $32^{3/5}$ **[2]**

(b) $100^{-1/2}$ **[1]**

(c) $8^{-2/3}$ **[2]**

3. [Maximum mark: 5] *Difficulty:* 2/5

Solve for x.

(a) $2^{3x} = \dfrac{1}{8}$ **[2]**

(b) $5^{2x+1} = 125$ **[3]**

4. [Maximum mark: 6] *Difficulty:* 2/5

Consider $\log_2 6 + \log_2 10 - \log_2 15$.

(a) Express this as a single logarithm. **[3]**

(b) Hence evaluate it exactly. **[3]**

5. [Maximum mark: 6] *Difficulty:* 2/5

Given that $\log_c 7 = s$ and $\log_c 2 = t$, express the following in terms of s and t.

(a) $\log_c 28$ **[3]**

(b) $\log_c 3.5$ **[3]**

6. [Maximum mark: 7] *Difficulty: 3/5*

Consider the equation $6^{2x-3} = 20$.

(a) Solve the equation, giving your answer in exact form in terms of a logarithm. **[4]**

(b) Hence write down the value of x correct to three significant figures. **[3]**

7. [Maximum mark: 7] *Difficulty: 3/5*

Solve the equation $\log_5(2x+3) - \log_5(x-1) = 1$.

(a) State the restriction on x required for the equation to be defined. **[2]**

(b) Solve the equation. **[5]**

8. [Maximum mark: 8] *Difficulty: 4/5*

Consider the equation $25^x - 6(5^x) + 5 = 0$.

(a) By using the substitution $y = 5^x$, show that $y^2 - 6y + 5 = 0$. **[3]**

(b) Hence find all values of x satisfying the original equation. **[5]**

9. [Maximum mark: 8] *Difficulty: 4/5*

(a) Prove that $\log_a(b^n) = n\log_a b$ for $a, b > 0$, $a \neq 1$ and $n \in \mathbb{Z}^+$, by writing b^n as a product of n factors of b. **[5]**

(b) Hence fully simplify $\log_2(32^5)$. **[3]**

10. [Maximum mark: 10] *Difficulty: 5/5*

Challenge question. Solve the equation

$$\log_x 2 + \log_2 x = \frac{5}{2}, \qquad x > 0,\ x \neq 1.$$

(a) By letting $u = \log_2 x$, express $\log_x 2$ in terms of u. **[3]**

(b) Hence form and solve a quadratic equation in u. **[4]**

(c) Find both corresponding values of x. **[3]**

Answer Key

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1. (a) $7^0 \times 7^3 = 7^3 = 343$. (b) $\dfrac{(5^2)^3}{5^4} = \dfrac{5^6}{5^4} = 5^2 = 25$.

2. (a) $32^{3/5} = (2^5)^{3/5} = 2^3 = 8$. (b) $100^{-1/2} = \dfrac{1}{10}$. (c) $8^{-2/3} = (2^3)^{-2/3} = 2^{-2} = \dfrac{1}{4}$.

3. (a) $2^{3x} = 2^{-3} \Rightarrow x = -1$. (b) $5^{2x+1} = 5^3 \Rightarrow 2x+1 = 3 \Rightarrow x = 1$.

4. (a) $\log_2 6 + \log_2 10 - \log_2 15 = \log_2\left(\dfrac{6\times 10}{15}\right) = \log_2 4$. (b) $\log_2 4 = 2$.

5. (a) $\log_c 28 = \log_c(4\times 7) = 2\log_c 2 + \log_c 7 = 2t + s$. (b) $\log_c 3.5 = \log_c\left(\dfrac{7}{2}\right) = s - t$.

6. (a) $(2x-3)\ln 6 = \ln 20 \Rightarrow x = \dfrac{1}{2}(3 + \log_6 20)$. (b) $x \approx 2.34$.

7. (a) Require $2x+3 > 0$ and $x-1 > 0$, so $x > 1$. (b) $\log_5\left(\dfrac{2x+3}{x-1}\right) = 1 \Rightarrow \dfrac{2x+3}{x-1} = 5 \Rightarrow 2x+3 = 5x-5 \Rightarrow 3x = 8 \Rightarrow x = \dfrac{8}{3}$, which satisfies $x > 1$.

8. (a) $25^x = (5^x)^2 = y^2$, so $y^2 - 6y + 5 = 0$. (b) $(y-1)(y-5) = 0 \Rightarrow y = 1$ or $y = 5$. $5^x = 1 \Rightarrow x = 0$; $5^x = 5 \Rightarrow x = 1$.

9. (a) $b^n = \underbrace{b \times b \times \cdots \times b}_{n \text{ factors}}$, so $\log_a(b^n) = \underbrace{\log_a b + \log_a b + \cdots + \log_a b}_{n \text{ terms}} = n\log_a b$, applying the product law repeatedly. (b) $\log_2(32^5) = 5\log_2 32 = 5(5) = 25$.

10. (a) $\log_x 2 = \dfrac{\ln 2}{\ln x} = \dfrac{1}{\ln x/\ln 2} = \dfrac{1}{u}$. (b) $u + \dfrac{1}{u} = \dfrac{5}{2} \Rightarrow 2u^2 - 5u + 2 = 0 \Rightarrow (2u-1)(u-2) = 0 \Rightarrow u = \dfrac{1}{2}$ or $u = 2$. (c) $u = 2 \Rightarrow x = 2^2 = 4$; $u = \dfrac{1}{2} \Rightarrow x = 2^{1/2} = \sqrt{2}$.