Exponents and Logarithms

Worksheet 2 — Set B

Mathematics: Analysis and Approaches HL

Instructions

- Answer all questions in the spaces provided. Show all working clearly: correct answers without working may not receive full marks.
- Unless told otherwise, give numerical answers exactly or correct to three significant figures.
- Questions are arranged in order of increasing difficulty. Question 10 on each worksheet is a *challenge question*, set beyond typical examination level.
- A calculator may be used unless a question indicates otherwise.

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1. [Maximum mark: 4] *Difficulty:* 1/5

Simplify, giving your answer as a single power.

(a) 5^{-2} **[2]**

(b) $(2^3)^{-1} \times 2^5$ **[2]**

2. [Maximum mark: 5] *Difficulty:* 1/5

Without using a calculator, evaluate each of the following.

(a) $27^{2/3}$ **[2]**

(b) $16^{-3/4}$ **[2]**

(c) 4^0 **[1]**

3. [Maximum mark: 5] *Difficulty:* 2/5

Solve for x.

(a) $4^x = \dfrac{1}{64}$ **[2]**

(b) $9^{x-2} = 3$ **[3]**

4. [Maximum mark: 6] *Difficulty:* 2/5

Consider $\log_5 18 - \log_5 2 + \log_5\left(\dfrac{25}{9}\right)$.

(a) Express this as a single logarithm. **[3]**

(b) Hence evaluate it exactly. **[3]**

5. [Maximum mark: 6] *Difficulty:* 2/5

Given that $\log_b 5 = m$ and $\log_b 2 = n$, express the following in terms of m and n.

(a) $\log_b 50$ **[3]**

(b) $\log_b 0.4$ **[3]**

6. [Maximum mark: 7] *Difficulty: 3/5*

Consider the equation $3^{4-x} = 11$.

(a) Solve the equation, giving your answer in exact form in terms of a logarithm. **[4]**

(b) Hence write down the value of x correct to three significant figures. **[3]**

7. [Maximum mark: 7] *Difficulty: 3/5*

Solve the equation $\log_3(x+5) + \log_3(x-1) = 2$.

(a) State the restriction on x required for the equation to be defined, and show that x satisfies $x^2 + 4x - 14 = 0$. **[3]**

(b) Hence solve the equation, rejecting any value of x that does not satisfy the required domain. **[4]**

8. [Maximum mark: 8] *Difficulty: 4/5*

Consider the equation $9^x - 4(3^x) - 45 = 0$.

(a) By using the substitution $y = 3^x$, show that $y^2 - 4y - 45 = 0$. **[3]**

(b) Hence find the value of x satisfying the original equation, explaining why only one root of the quadratic is valid. **[5]**

9. [Maximum mark: 8] *Difficulty: 4/5*

(a) Prove that $\log_a\left(\frac{1}{b}\right) = -\log_a b$, for $a, b > 0$, $a \neq 1$. **[4]**

(b) Hence, using the change of base formula, evaluate $\log_{1/2} 8$ exactly. **[4]**

10. [Maximum mark: 10] *Difficulty: 5/5*

Challenge question. Find all real solutions of the equation

$$x^{\log_2 x} = 16, \qquad x > 0.$$

(a) By taking $\log_2$ of both sides, show that $(\log_2 x)^2 = 4$. **[4]**

(b) Hence solve the resulting equation for $\log_2 x$. **[3]**

(c) State both values of x, and verify that each satisfies the original equation. **[3]**

Answer Key

1. (a) $5^{-2} = \frac{1}{25}$. (b) $(2^3)^{-1} \times 2^5 = 2^{-3} \times 2^5 = 2^2 = 4$.

2. (a) $27^{2/3} = (3^3)^{2/3} = 3^2 = 9$. (b) $16^{-3/4} = (2^4)^{-3/4} = 2^{-3} = \frac{1}{8}$. (c) $4^0 = 1$.

3. (a) $4^x = 4^{-3} \Rightarrow x = -3$. (b) $9^{x-2} = 3^{2(x-2)} = 3^1 \Rightarrow 2(x-2) = 1 \Rightarrow x = 2.5$.

4. (a) $\log_5 18 - \log_5 2 + \log_5\left(\frac{25}{9}\right) = \log_5\left(\frac{18}{2}\cdot\frac{25}{9}\right) = \log_5\left(9\cdot\frac{25}{9}\right) = \log_5 25$. (b) $\log_5 25 = 2$.

5. (a) $\log_b 50 = \log_b(2\times 25) = \log_b 2 + 2\log_b 5 = n + 2m$. (b) $\log_b 0.4 = \log_b\left(\frac{2}{5}\right) = n - m$.

6. (a) $(4-x)\ln 3 = \ln 11 \Rightarrow x = 4 - \log_3 11$. (b) $x \approx 1.82$.

7. (a) Require $x+5 > 0$ and $x-1 > 0$, so $x > 1$. $\log_3[(x+5)(x-1)] = 2 \Rightarrow (x+5)(x-1) = 9 \Rightarrow x^2+4x-5 = 9 \Rightarrow x^2+4x-14 = 0$. (b) $x = \frac{-4\pm\sqrt{16+56}}{2} = -2\pm 3\sqrt{2}$. Since $x > 1$, reject $x = -2-3\sqrt{2}$; so $x = -2+3\sqrt{2} \approx 2.24$.

8. (a) $9^x = (3^x)^2 = y^2$, so $y^2 - 4y - 45 = 0$. (b) $(y-9)(y+5) = 0 \Rightarrow y = 9$ or $y = -5$. Since $y = 3^x > 0$ for all real x, reject $y = -5$. $3^x = 9 \Rightarrow x = 2$.

9. (a) $\log_a\left(\frac{1}{b}\right) = \log_a 1 - \log_a b = 0 - \log_a b = -\log_a b$. (b) $\log_{1/2} 8 = \frac{\ln 8}{\ln(1/2)} = \frac{3\ln 2}{-\ln 2} = -3$.

10. (a) Taking $\log_2$: $\log_2(x^{\log_2 x}) = (\log_2 x)(\log_2 x) = (\log_2 x)^2$, and $\log_2 16 = 4$, so $(\log_2 x)^2 = 4$. (b) $\log_2 x = \pm 2$. (c) $x = 2^2 = 4$ or $x = 2^{-2} = \frac{1}{4}$. Check: $4^{\log_2 4} = 4^2 = 16$; $\left(\frac{1}{4}\right)^{\log_2(1/4)} = \left(\frac{1}{4}\right)^{-2} = 16$.