Exponents and Logarithms

Worksheet 1 — Set A

Mathematics: Analysis and Approaches HL

Instructions

- Answer all questions in the spaces provided. Show all working clearly: correct answers without working may not receive full marks.
- Unless told otherwise, give numerical answers exactly or correct to three significant figures.
- Questions are arranged in order of increasing difficulty. Question 10 on each worksheet is a *challenge question*, set beyond typical examination level.
- A calculator may be used unless a question indicates otherwise.

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1. [Maximum mark: 4] *Difficulty:* 1/5

Simplify, giving your answer as a single power.

(a) $2^3 \times 2^5$ **[2]**

(b) $\dfrac{(3^4)^2}{3^5}$ **[2]**

2. [Maximum mark: 5] *Difficulty:* 1/5

Without using a calculator, evaluate each of the following.

(a) $8^{2/3}$ **[1]**

(b) $25^{-1/2}$ **[2]**

(c) $\left(\dfrac{1}{16}\right)^{3/4}$ **[2]**

3. [Maximum mark: 5] *Difficulty:* 2/5

Solve for x.

(a) $3^x = 81$ **[2]**

(b) $2^{x+1} = 32$ **[3]**

4. [Maximum mark: 6] *Difficulty:* 2/5

Consider $\log_2 20 + \log_2 4 - \log_2 5$.

(a) Express this as a single logarithm. **[3]**

(b) Hence evaluate it exactly. **[3]**

5. [Maximum mark: 6] *Difficulty:* 2/5

Given that $\log_a 2 = p$ and $\log_a 3 = q$, express the following in terms of p and q.

(a) $\log_a 12$ **[3]**

(b) $\log_a\left(\dfrac{9}{2}\right)$ **[3]**

6. [Maximum mark: 7] *Difficulty:* 3/5

Consider the equation $5^{2x-1} = 7$.

(a) Solve the equation, giving your answer in exact form in terms of a logarithm. **[4]**

(b) Hence write down the value of x correct to three significant figures. **[3]**

7. [Maximum mark: 7] *Difficulty:* 3/5

Solve the equation $\log_2 x + \log_2(x-2) = 3$, where $x > 2$.

(a) Show that x satisfies $x^2 - 2x - 8 = 0$. **[3]**

(b) Hence solve the equation, rejecting any value of x that does not satisfy the given domain. **[4]**

8. [Maximum mark: 8] *Difficulty:* 4/5

Consider the equation $4^x - 6(2^x) + 8 = 0$.

(a) By using the substitution $y = 2^x$, show that $y^2 - 6y + 8 = 0$. **[3]**

(b) Hence find all values of x satisfying the original equation. **[5]**

9. [Maximum mark: 8] *Difficulty:* 4/5

(a) Prove the change of base formula $\log_a b = \dfrac{\ln b}{\ln a}$, for $a, b > 0$ and $a \neq 1$. **[5]**

(b) Hence evaluate $\log_7 50$, correct to three significant figures. **[3]**

10. [Maximum mark: 10] *Difficulty:* 5/5

Challenge question. Find the values of x and y, both greater than zero, which satisfy the simultaneous equations

$$\log_2 x + \log_4 y = 4, \qquad \log_4 x + \log_2 y = \frac{7}{2}.$$

(a) By letting $a = \log_2 x$ and $b = \log_2 y$, express both equations as linear equations in a and b. **[4]**

(b) Solve the resulting system to find a and b. **[3]**

(c) Hence find the values of x and y. **[3]**

Answer Key

1. (a) $2^3 \times 2^5 = 2^8 = 256$. (b) $\dfrac{(3^4)^2}{3^5} = \dfrac{3^8}{3^5} = 3^3 = 27$.

2. (a) $8^{2/3} = (2^3)^{2/3} = 2^2 = 4$. (b) $25^{-1/2} = \dfrac{1}{5}$. (c) $\left(\dfrac{1}{16}\right)^{3/4} = (2^{-4})^{3/4} = 2^{-3} = \dfrac{1}{8}$.

3. (a) $3^x = 3^4 \Rightarrow x = 4$. (b) $2^{x+1} = 2^5 \Rightarrow x+1 = 5 \Rightarrow x = 4$.

4. (a) $\log_2 20 + \log_2 4 - \log_2 5 = \log_2\left(\dfrac{20\times 4}{5}\right) = \log_2 16$. (b) $\log_2 16 = 4$.

5. (a) $\log_a 12 = \log_a(4\times 3) = 2\log_a 2 + \log_a 3 = 2p+q$. (b) $\log_a\left(\dfrac{9}{2}\right) = \log_a 9 - \log_a 2 = 2q - p$.

6. (a) $(2x-1)\ln 5 = \ln 7 \Rightarrow x = \dfrac{1}{2}(1+\log_5 7)$. (b) $x \approx 1.10$.

7. (a) $\log_2[x(x-2)] = 3 \Rightarrow x(x-2) = 8 \Rightarrow x^2 - 2x - 8 = 0$. (b) $(x-4)(x+2) = 0 \Rightarrow x = 4$ or $x = -2$; since $x > 2$, reject $x = -2$, so $x = 4$.

8. (a) $4^x = (2^x)^2 = y^2$, so $y^2 - 6y + 8 = 0$. (b) $(y-2)(y-4) = 0 \Rightarrow y = 2$ or $y = 4$. $2^x = 2 \Rightarrow x = 1$; $2^x = 4 \Rightarrow x = 2$.

9. (a) Let $\log_a b = x$, so $a^x = b$. Taking ln of both sides: $x\ln a = \ln b \Rightarrow x = \dfrac{\ln b}{\ln a}$. (b) $\log_7 50 = \dfrac{\ln 50}{\ln 7} \approx \dfrac{3.912}{1.946} \approx 2.01$.

10. (a) $\log_4 y = \dfrac{b}{2}$ and $\log_4 x = \dfrac{a}{2}$, so the equations become $2a+b = 8$ and $a+2b = 7$. (b) Doubling the first equation: $4a+2b = 16$; subtracting the second: $3a = 9 \Rightarrow a = 3$, so $b = 8-2(3) = 2$. (c) $x = 2^3 = 8$, $y = 2^2 = 4$.